Neutrino Oscillations in Brans–Dicke Theory of Gravity

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Abstract

Flavor oscillations of neutrinos are analyzed in the framework of Brans–Dicke theory of gravity. We find a shift of quantum mechanical phase of neutrino proportional to $G_N \Delta m^2$ and depending on the parameter ω. Consequences on atmospheric, solar and astrophysical neutrinos are discussed.

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1 Introduction

Among all alternative theories of gravity, the Brans–Dicke (BD) theory [1] provides the most natural generalization of General Relativity. It can be thought of as a minimal extension of Einstein theory in which Mach's principle and Dirac's large number hypothesis (see, for example, [2]) are properly accommodated by means of a nonminimal coupling between the geometry and a scalar field ϕ, the BD scalar. The scalar field rules dynamics together with geometry and, furthermore, induces a variation of the gravitational *coupling* with time and space through the relation $G_{eff} = 1/\phi$. The gravitational constant G_N is recovered in the limit $\phi \to constant$. Some recent experiments [3] seem to confirm a variation of the Newton constant on astrophysical and cosmological sizes and time scale.

The effective action describing the interaction of the scalar field ϕ nonminimally coupled with the geometry and the ordinary matter is given by [1]

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[\phi R - \omega\frac{\partial_\mu\phi\partial^\mu\phi}{\phi} + \frac{16\pi}{c^4}\mathcal{L}_m\right], \tag{1.1}$$

where R is the scalar curvature, $\mathcal{L}_m$ is the matter contribution in the total Lagrangian density. The constant ω is determined by observations and its value can be constrained by classical tests of General Relativity. The consequences of BD action (1.1) have been analyzed for the light deflection, the relativistic perihelion rotation of Mercury, and the time delay experiment, resulting in reasonable agreement with all available observations thus far provided $\omega \geq 500$ [4]. On the other hand, bounds on the anisotropy of the microwave background radiation give the upper limit $\omega \leq 30$ [5]. Einstein's theory is recovered for $\omega \to \infty$. In this limit, the BD theory becomes indistinguishable from General Relativity in all its predictions.

Understanding if the BD theory of gravity may be considered as the right generalization of Einstein gravity and, as a consequence, how it affects physical phenomena is an important matter. In this paper we will face this issue by considering neutrino oscillations, calculating, in particular, the contribution to the quantum mechanical phase mixing induced by the non–standard coupling between the geometry and the scalar field. As we will see, such a correction does depend on the value of the parameter ω.

It is well known that the problem of neutrino oscillations is still open, and the research of new effects in which they could manifest is one of the main task of the today physics. For this reason, the quantum mechanical phase of neutrinos propagating in gravitational field (usually the Schwarzschild or Kerr field) has been recently discussed by several authors (see [6]–[14] and references therein), also in view of astrophysical consequences.

More controversial is the debate concerning the red-shift of flavor oscillation clocks, given by a term proportional to

$$\frac{G_N\Delta m^2 M}{\hbar E}\,\log\frac{r_B}{r_A}\,, \tag{1.2}$$

first derived by Ahluwalia and Burgard [6] in the framework of the weak gravitational field of a star, with mass M. Here Δm^2 is the mass–squared difference, $\Delta m^2 = |m_2^2 - m_1^2|$, E the neutrino energy, r_A and r_B the points where neutrinos are created and detected, respectively. They also suggest that the oscillation phase (1.2) might have a significant effect for supernova explosions due to the extremely large fluxes of neutrinos produced with different energies, corresponding to the flavor states.

This result has been confirmed in the paper by Grossman and Lipkin [9], and it has been also derived by Konno and Kasai [11] under the assumption that the radial momentum of neutrinos is constant along the trajectory of the neutrino, strengthening the correctness of the Ahluwalia–Burgard arguments. Nevertheless, assuming that the neutrino energy is constant along the trajectory, Konno and Kasai show that the term (1.2) is cancelled out, recovering in such a way the result of Refs. [12, 13].

Without pretending to solve or face here this controversy, which goes beyond our aim, this paper is a straightforward extension of the calculations of Ref. [12] in the framework of BD theory. In [12], the neutrino oscillation formula in a gravitational field is based on the *covariant form* of the quantum phase that arises due to the assumed mixing of massive neutrino. The result (i.e. the cancellation of $G_N \Delta m^2$ term) is the same of Ref. [13], but it is derived without invoking the assumption that underlying mass eigenstates are emitted at different time. We find that the scalar field in (1.1) nonminimally coupled to the scalar curvature induces a red-shift of flavor oscillation clocks in the quantum dynamical phase, which is proportional to

$$\frac{G_N \Delta m^2}{E} \frac{1}{2+\omega} \log \frac{r_B}{r_A} \,. \tag{1.3}$$

It vanishes in the limit $\omega \to 0$[1]. Eq. (1.3) can be seen, in some sense, as a further test, in addition to the standard ones above discussed, for establishing the validity (or not) of the BD theory.

The layout of this paper is the following. In Sect. 2 we shortly recall the Schwarzschild–like solution coming from BD field equations, which describe the static and stationary gravitational field generated by a mass M, and the corresponding expressions in the weak field approximation (for details, see the paper [1]). Sect. 3 is devoted to the calculation of the quantum mechanical phase for propagating neutrinos in the BD geometry. Conclusions are drawn in Sect. 4.

[1]The extension of the paper [6] (or [9]) to the BD theory does not give appreciable correction to the quantum dynamical phase. In fact, the corrective factor is of the form

$$\frac{3+2\omega}{2(2+\omega)}, \tag{1.4}$$

which is ~ 1 for $\omega \sim 500$ and $\omega \leq 30$.

2 Static Spherically Symmetric Field in BD Theory

Variation of the action (1.1) with respect to the tensor metric $g_{\mu\nu}$ and the scalar field ϕ yields to the field equations [1]

$$R_{\mu\nu} - \frac{1}{2}\,R = \frac{8\pi}{c^4\phi}\,T_{\mu\nu} + \frac{\omega}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right) + \frac{1}{\phi}\left(\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi\right) \tag{2.1}$$

for the geometric part, and

$$\frac{2\omega}{\phi}\,\Box\phi - \frac{\omega}{\phi^2}\phi_{,\mu}\phi^{,\mu} + R = 0 \tag{2.2}$$

for the scalar field. $\Box$ is the usual d'Alembert operator in curved space–time and $T_{\mu\nu}$ is the momentum–energy tensor of matter. The line element describing a static and isotropic geometry is expressed as

$$ds^2 = -e^{2\alpha}dt^2 + e^{2\beta}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]\,, \tag{2.3}$$

where the functions α and β depend on the radial coordinate r. The general solution in the vacuum is given by

$$e^{2\alpha} \;=\; e^{2\alpha_0}\left[\frac{1-B/r}{1+B/r}\right]^{2/\lambda}\,, \tag{2.4}$$

$$e^{2\beta} \;=\; e^{2\beta_0}\left(1+\frac{B}{r}\right)^4\left[\frac{1-B/r}{1+B/r}\right]^{2(\lambda-C-1)/\lambda}\,, \tag{2.5}$$

$$\phi \;=\; \phi_0\left[\frac{1-B/r}{1+B/r}\right]^{-C/\lambda}\,, \tag{2.6}$$

where the constants, appropriately chosen, are given by

$$\lambda = \sqrt{\frac{2\omega+3}{2(\omega+2)}}\,, \quad C \cong -\frac{1}{2+\omega}\,, \quad \alpha_0 = 0 = \beta_0\,, \tag{2.7}$$

$$\phi_0 = \frac{4+2\omega}{G_N(3+2\omega)}\,, \quad B = \frac{M}{2c^2\phi_0}\sqrt{\frac{2\omega+4}{2\omega+3}}\,.$$

In the weak field approximation, the components of the tensor metric, $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$, reduces to the form [1]

$$g_{00} \;\simeq\; -1 + \frac{2M\phi_0^{-1}}{c^2 r}\frac{4+2\omega}{3+2\omega}\,, \tag{2.8}$$

$$g_{ii} \;\sim\; 1 + \frac{2M\phi_0^{-1}}{c^2 r}\frac{2+2\omega}{3+2\omega}\,, \quad i = 1,2,3, \tag{2.9}$$

$$g_{0i} \;=\; 0\,, \qquad g_{ij} = 0\,, \quad i \neq j\,, \tag{2.10}$$

$$\phi \;=\; \phi_0 + \frac{2M}{c^2 r}\frac{1}{3+2\omega}\,. \tag{2.11}$$

As discussed in Introduction, the weak–field solutions (2.8)–(2.11) have been analyzed for gravitational red–shift, the deflection of light and perihelion of Mercury (to be more precise, the last one requires an approximation up to the second order in M/r). In the next Section we will investigate the phenomenological consequences of BD solutions (2.8)–(2.10) on neutrinos propagating in such a geometry.

3 Neutrino Oscillations in BD Geometry

The effects of gravitational fields on the quantum mechanical neutrino oscillation phases have been analyzed in the semi–classical approximation, in which the action of a particle is considered as a quantum phase [15]. In calculating such effects induced by BD geometry, we will use the same approximation.

A particle propagating in a gravitational field from a point A to a point B, changes its quantum mechanical phase according to the relation [15]

$$\Phi = \frac{1}{\hbar}\int_A^B m ds = \frac{1}{\hbar}\int_A^B p_\mu dx^\mu \,, \tag{3.1}$$

where $p_\mu = m g_{\mu\nu}(dx^\nu/ds)$ is the four–momentum of the particle and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Following Ref. [12], the quantum mechanical phase becomes

$$\Phi = \frac{1}{\hbar}\int_{r_A}^{r_B}\left[E\frac{dt}{dr} - p_r\right] dr \,. \tag{3.2}$$

Inserting the momentum of the particle, coming from the shell–condition, $g_{\mu\nu}p^\mu p^\nu = m^2$,

$$p_r = e^{\beta-\alpha}\sqrt{E^2 - m^2 e^{2\alpha}} \tag{3.3}$$

into Eq. (3.2), and using the fact that $dt/dr = e^{\beta-\alpha}$, one gets a difference phase $\Delta\Phi$ given by

$$\Delta\Phi = \frac{\Delta}{2\hbar E}\int_{r_A}^{r_B}\left(1+\frac{B}{r}\right)^2\left(\frac{1-B/r}{1+B/r}\right)^{(\lambda-C)/\lambda}\left[E - \sqrt{E^2 - m^2\left(\frac{1-B/r}{1+B/r}\right)^{2/\lambda}}\right] dr \,. \tag{3.4}$$

By using the weak field approximation, Eqs. (2.8)–(2.10), one can separate out the *gravitational* contribution to the neutrino oscillation phase, so that Eq. (3.4) can be cast in the form

$$\Delta\Phi = \Delta\Phi_0 + \Delta\Phi_\omega \,, \tag{3.5}$$

where (restoring the constants c and $\hbar$)

$$\Delta\Phi_0 = \frac{\Delta m^2 c^3}{2E\hbar}(r_B - r_A) \,, \tag{3.6}$$

which represents the standard phase of neutrino oscillations, and

$$\Delta\Phi_\omega = \frac{\Delta m^2 c}{2\hbar E}\frac{G_N M}{2+\omega}\log\frac{r_B}{r_A}\,. \tag{3.7}$$

In deriving Eqs. (3.6) and (3.7), we have considered ultra–relativistic neutrinos, $E >> m$, where E is interpreted as the energy at the infinite (see [12] for details). The integration has been performed along the light–ray trajectory where E is constant.

It is convenient to rewrite the phases (3.6) and (3.7) in the following way

$$\Delta\Phi_0 \approx 2.5\cdot 10^3 \frac{\Delta m^2}{\mathrm{eV}^2/c^4}\,\frac{\mathrm{MeV}}{E}\,\frac{r_B - r_A}{\mathrm{Km}}\,, \tag{3.8}$$

and

$$\Delta\Phi_\omega \approx 3.5\cdot 10^3\,\frac{1}{2+\omega}\,\frac{\Delta m^2}{\mathrm{eV}^2/c^4}\,\frac{\mathrm{MeV}}{E}\,\frac{M}{M_\odot}\,\log\frac{r_B}{r_A}\,, \tag{3.9}$$

where $M_\odot$ is the solar mass. Estimations of the difference phases (3.8) and (3.9) are carried out for solar, atmospheric and astrophysical neutrinos. To this end, we will introduce the ratio q defined as

$$q = \frac{\Delta\Phi_\omega}{\Delta\Phi_0} \approx 1.5\,\frac{1}{2+\omega}\,\frac{M}{M_\odot}\,\frac{\log(r_B/r_A)}{(r_B - r_A)/\mathrm{Km}}\,. \tag{3.10}$$

q does not depend on the squared–mass difference Δm^2 and on the neutrino energy E. For solar neutrinos, we use the following values: $M \sim M_\odot$, $r_A \sim r_{Earth} \sim 6.3\cdot 10^3$Km, and $r_B \sim r_A + D$, where $D \sim 1.5\cdot 10^8$Km is the Sun–Earth distance. Eq. (3.10) gives the result

$$q \sim 10^{-8}\,\frac{1}{2+\omega}\,, \tag{3.11}$$

which is an irrelevant correction to the difference phase (3.8). Analogous conclusion holds for atmospheric neutrinos.

Concerning the astrophysical neutrinos, the effect could be more relevant and could be measured by terrestrial experiments. In fact, setting $r_B = \alpha r_A$, $1 < \alpha \leq \infty$ and using the typical values of neutron stars, $M \sim 1.4 M_\odot$ and radius $r_A \sim 10$Km as in Ref. [6], we get

$$q \sim \frac{0.2}{2+\omega}\,\frac{\log\alpha}{\alpha - 1}\,. \tag{3.12}$$

Till now, our analysis has been done for radially propagating neutrinos. In the case of motion transverse to the radial propagation and near to the detection point r_A we have, following [6],

$$\Delta\Phi_\omega^\perp = \frac{\Delta m^2 c}{2\hbar E}\frac{G_N M}{2+\omega}\frac{r_B - r_A}{r_A} \approx 3.5\cdot 10^3\,\frac{1}{2+\omega}\,\frac{\Delta m^2}{\mathrm{eV}^2}\,\frac{\mathrm{MeV}}{E}\,\frac{M}{M_\odot}\,\frac{r_B - r_A}{r_A}\,. \tag{3.13}$$

Then, the ratio between the difference phases (3.13) and (3.8) is

$$q^{\perp} = \frac{\Delta\Phi_{\omega}^{\perp}}{\Delta\Phi_0} \approx 1.5\,\frac{1}{2+\omega}\,\frac{M}{M_{\odot}}\,\frac{\mathrm{Km}}{r_A}\,. \tag{3.14}$$

For the numerical constants corresponding to Sun and Earth, we have

$$q_{Sun}^{\perp} \sim \frac{1.5\cdot 10^{-5}}{2+\omega}\,, \qquad q_{Earth}^{\perp} \sim \frac{5\cdot 10^{-10}}{2+\omega}\,. \tag{3.15}$$

Using the above values for a neutron star, Eq. (3.14) gives the result

$$q^{\perp} \sim \frac{0.2}{2+\omega}\,. \tag{3.16}$$

As discussed in Introduction, experimental data imply that the parameter ω can assume the value $\omega \geq 500$. For the lower limit, one gets from Eqs. (3.12) and (3.16),

$$q \sim 4\cdot 10^{-4}\,\frac{\log\alpha}{\alpha-1}\,, \qquad q^{\perp} \sim 4\cdot 10^{-4}\,, \tag{3.17}$$

giving a correction of the 0.01 percent. Values of $\omega \leq 30$, coming from the anisotropy of microwave background radiation, allow to get corrections of few percents, as one can immediately derive from Eqs. (3.12) and (3.16). Such contributions to the quantum mechanical phase of neutrinos, are very significant and could be considered as a test for establishing the validity of BD theory.

4 Conclusions

In this paper, we have analyzed neutrino oscillation phenomena in the framework of BD theory. We have derived a correction to the standard difference phase of the order $G_N\Delta m^2$, which vanishes in the limit $\omega \to \infty$, when the BD theory reduces to General Relativity.

Estimation of such a correction has been carried out assuming for the parameter ω the values $\omega \sim 500$ and $\omega \leq 30$. Such values may be relaxed considerably with the advances in technology associated with astronomical observations and astrophysical experiments, making our corrections as a mean to discern between BD theory and Einstein's theory, in addition to the ones discussed in the Introduction.

Nevertheless, BD is a particular case of scalar tensor–theories where one assumes that matter acts as source of scalar field ϕ, which generates the curvature of space–time associated to the metric. The strength of the coupling between the scalar field and gravity is determined, in these theories, by the function $\omega(\phi)$, which is constant in the BD theory. Besides, a self–interaction potential $V(\phi)$ can be also introduced, generalizing in such a way dynamics of the field.

The dependence of the parameter ω on ϕ could have the property that, at the present epoch, and in weak field situations, the value of the scalar field ϕ_0 is such that ω is very large, leading to theories almost identical to General Relativity today, but for past or future values of ϕ, as in strong field regimes as for neutron stars, ω could take values that would lead to significant differences from General Relativity. In this sense, scalar–tensor theories are richer than BD theory and could play a relevant role in the neutrino oscillation physics (and in Pound–Rebka or COW experiments, as well as in atomic systems in linear superposition of different energy eigenstates). This because the variability of the parameter ω implies that, in some epoch, its value could be very small, and, in such a way, a correction to the quantum mechanical phase of 10% can be obtained (in this particular case, the factor (1.4) reduces the Ahluwalia–Burgard result to 15%, instead of 20% as derived in Ref. [6]). In a forthcoming paper we will face these issues.

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